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Stable ballistic coefficient forms of spatial agglomerations of microparticles in the gravitational-repulsive field of binary stellar systems☆

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ABSTRACT

A new physically clearer analysis of the stability of the spatial positions of relative equilibrium (coplanar libration points) of micrometeorite particles or particles of gas-dust clouds in the field of two gravitating and radiating stars is presented based on a modified version of the restricted circular three-body problem. It is shown that, unlike the classical version of the problem, there are complete families of libration points, in which stable cloud agglomerations of particles are formed in a plane orthogonal to the orbital plane of the main bodies (stars). The evolution of these agglomerations is investigated as a function of a certain generalized parameter, characterizing the gravitational-repulsive field of a stellar pair.

One of the most interesting and physically adequate applications of the restricted three-body problem is its use to investigate the motions of micrometeorite particles or particles of gas-dust clouds in the gravitational-repulsive field of a binary stellar system, when the particles, in addition to the gravitational action from the stars, also experience a considerable light pressure force (which is greater the smaller the dimensions of the particle) originating from these. Interest in this problem, first formulated by Radziyevskii,[1] and called the photogravitational problem, has increased considerably recently and primarily in connection with investigations of the positions of relative equilibrium in a system of coordinates rotating together with the stars, similar to the well-known libration points in the classical three-body problem. These equilibrium positions (including an investigation of their stability) have been investigated in a number of papers,[2] but a fundamental difference between the case considered and the classical problem was not taken into account, namely, whereas in the classical problem the positions of all five libration points (two triangular and three collinear) in no way depend on the dimensions and mass of the passively gravitating points, in the photogravitational problem the position of a libration point is determined to an equal extent both by the parameters of the gravitational-repulsive field and by the mass and dimensions of the particle itself. This enables us to speak of the occurrence of complete families of libration points and, consequently, of the cloud agglomeration of micrometeorite particles or particles of gas-dust clouds filling them. Particular clarity can be attributed to the results obtained if the region of stability of these agglomerations is constructed directly in configuration space, rather than in parametric space, as was done previously.[2] The problem of stable forms of cloud agglomerations in the orbital plane of the main bodies was solved precisely in this formulation.[3]

The purpose of this paper is to use the same approach to investigate the stability of cloud agglomerations of particles in the plane orthogonal to the orbital plane of the motion of the main bodies and the corresponding coplanar solutions of the circular photogravitational three-body problem.

1. Families of spatial positions of relative equilibrium

Suppose *Oxyz* is a barycentric system of rectangular coordinates, rotating uniformly around the z axis; point masses S_1 and S_2, representing a specified stellar pair with masses M_1 and M_2 respectively, are situated on the x axis. The mass reduction factors Q_1 and Q_2 of the bodies S_1 and S_2, characterizing the weakening of the gravitational action of these bodies on a particle due to light repulsion, can be represented in the form

$$Q_i = (1 - F_p/F_g)_i = 1 - C_i\sigma/(fM_i), \quad i = 1, 2 \tag{1.1}$$

Here F_g and F_p are the gravitational and repulsive forces acting on the particle from one of the main bodies, respectively, C_i is the radiation power of the body S_i, σ is the ballistic coefficient of a particle, equal to the ratio of the characteristic cross-section area of a particle to its mass, and f is the gravitational constant.

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It was shown in Refs. 4–6 that a passive gravitational particle can be in a position of relative equilibrium in the xz plane if the following conditions are satisfied

$$Q_1 = xR_1^3/(1-\mu), \quad Q_2 = -xR_2^3/\mu \tag{1.2}$$

where R_1 and R_2 are the distances of the particle to the point masses S_1 and S_2 respectively and $1-\mu$ and μ are their dimensionless masses.

It can be seen that the reduction factors depend not only on the parameters of the gravitational-repulsive field, but also on the parameters of the particles themselves in the form of their ballistic coefficient σ, which increases as the absolute dimensions of the particles decrease. This also enables us to allow of the possibility of the existence of whole families of positions of relative equilibrium, which is impossible when there is no light repulsion.

It can also be seen that physically permissible values of Q_i are defined by the inequalities $Q_i \le 1$ (when $Q_i = 1$ there is no radiation and, consequently, no light pressure). When $Q_i > 0$ gravitation predominates over repulsion, when $Q_i = 0$ it is completely neutralized by it, and when $Q_i < 0$ a single repulsion remains. Since the ballistic coefficient is determined solely by the parameters of the particle itself, it follows from relations (1.1) that for any chosen stellar pair S_1 and S_2 the reduction factors are connected by the relation

$$\frac{1-Q_1}{1-Q_2} = k, \quad k = \frac{C_1/M_1}{C_2/M_2} \tag{1.3}$$

which, in the Q_1, Q_2 plane, represents a bundle of straight lines passing through the point $Q_1 = Q_2 = 1$.

The coefficient k, which distinguishes one of the straight lines of this bundle, and the mass parameter μ completely characterize the gravitational-repulsive field of the chosen stellar pair. Particles with a different ballistic coefficient σ, which increases with distance from the point $Q_1 = Q_2 = 1$, which, as can easily be seen, occurs when the absolute dimensions of the particle decrease, correspond to different points of one of the straight lines (1.3).

Hence, the particular solutions considered (the coplanar libration points) represent three-parameter families of positions of relative equilibrium, in which two parameters (μ and k) completely characterize the gravitational-repulsive field of the chosen stellar pair, while the third (σ) defines the position of a particle on the curve, representing the geometrical position of points of the family.

Note that, in the majority of previous papers,[2] devoted to the case considered, the reduction factors were either considered to be independent parameters or were assumed to be connected by a relation different from (1.3), to which it is difficult to give any physical explanation.[4–6] This formulation of the problem led to the incorrect conclusion that only two or only two pairs of coplanar libration points, symmetrical about the x axis, exist,[4,6] whereas, according to the above considerations, for any fixed stellar pair there can be an innumerable set of them.

We obtain the equation of the curve defining the geometrical position of the libration points considered, taking into account the established relation between the reduction factors, by eliminating in relation (1.3) Q_1 and Q_2, defined by formulae (1.2), in which

$$R_1^2 = (x+\mu)^2 + z^2, \quad R_2^2 = (x-1+\mu) + z^2$$

The relation between z and x obtained as a result of this has a quite complex form and does not allow of an explicit expression of one in terms of the other. The curves $z = z(x)$ for different values of μ and k corresponding to this relation are shown in Fig. 1. Each such curve also represents the required form of the cloud agglomeration of particles for a fixed stellar pair. However, not all the points of these curves correspond to stable positions of relative equilibrium.

As can be seen from Eq. (1.2), Q_1 and Q_2 must necessarily have different signs, whence it follows from elementary physical con-

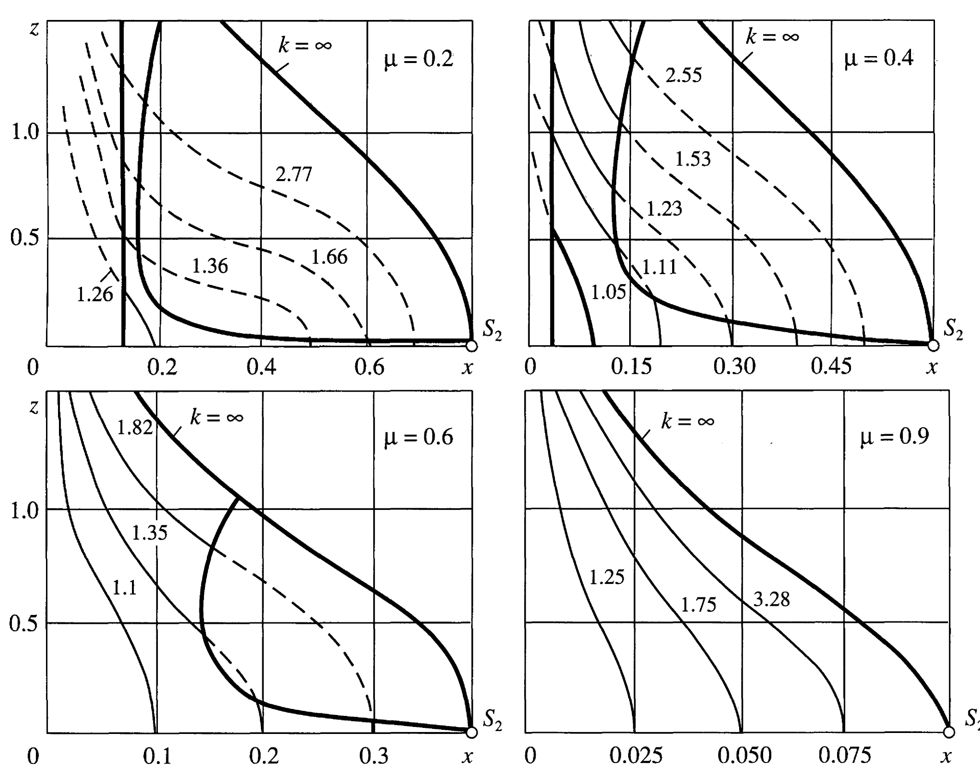

Fig. 1.

siderations that the equilibrium positions considered can only be situated on one side of the z axis, namely, the one where the body with a negative value of the reduction factor is situated. Suppose this is the body S_2 with a dimensionless mass μ and a reduction factor $Q_2 < 0$. Consequently, the abscissae of all the libration points in this case will be positive, while all the possible values of k, as follows from relations (1.3), satisfy the inequality $0 \le k \le 1$. When $k = 1$, in which case $Q_1 = Q_2 = 0$, we have an innumerable set of indifferent equilibrium positions, filling the whole z axis, and when $k = 0$ ($Q_1 = 1$, $Q_2 < 0$) we have the case when the body S_1 does not radiate and all the equilibrium positions lie on the curve

$$xR_1^3 = 1 - \mu$$

which is symmetrical about the x axis and represents simultaneously the limit of possible equilibrium positions for all possible values of the parameter k.

Note that if μ is allowed to take any values from the interval $0 < \mu < 1$, rather than traditionally assumed to be subject to the inequality $\mu < ½$, then the abscissae of the coplanar libration points can always be assumed to be positive (when $\mu < ½$ only the action of the lesser mass is reduced to repulsion, and when $\mu < ½$ only the larger mass).

2. Selection of the stable families

Since the potential energy of the system for the equilibrium positions considered has no minimum for any values of x and z, here, as also in the case of collinear and triangular libration points, we can only count on gyroscopic stabilization, the conditions of which can be derived from a consideration of the linearized equations of perturbed motion. The stability region, defined by these conditions, are best constructed in configuration space, i.e., in the xz plane, rather than in the space of the parameters Q_1 and Q_2, as was done earlier in Ref. 4.

As we know from the general theory of Hamiltonian systems (to which the problem in question belongs), the conditions for such stabilization imply complete Birkhoff stability, i.e., which is also preserved when non-linear terms of as high but finite order as desired are taken into account in the equations of perturbed motion, with the exception, possibly, of values of the parameters corresponding to second, third and fourth order resonances (higher order resonances may lead to instability only in the case of a very high degree of degeneration of the normal form of the Hamiltonian). Note that non-satisfaction of the conditions of gyroscopic stabilization denotes strict Lyapunov instability.

To obtain these conditions it is necessary, in the linearized equations of perturbed motion (in this case representing a sixth-order system), to eliminate the reduction factors Q_1 and Q_2 using formulae (1.2). Then, the inequalities which are the conditions for gyroscopic stabilization (obtained from the requirement that there should be no real parts in the roots of the corresponding characteristic bicubic equation), will contain coordinates of the libration points and the parameter μ. These regions in the xz plane (they are symmetrical about the x axis) were previously constructed for certain values of μ in Refs. 5 and 6, and for other values of μ ($\mu = 0.4$, 0.6 and 0.9) they are shown in Fig. 1 (the limits of the regions are represented by the heavy curves). The curves corresponding to positions of relative equilibrium for fixed values of the parameter k are shown and enable one to judge which parts of these curves correspond to stable families and which do not (the parts of the curves for unstable families are denoted by dashes). It can be seen from these results that the stability region expands as the parameter μ increases (i.e., as the mass of the more strongly radiating body increases) and it is not just the parts of the curves of the families considered, corresponding to values of k close to unity for small μ, that fall within it.

The above analysis therefore enables us to obtain a typically distinct and geometrically clearer pattern of the formation of stable forms of cloud agglomeration of particles than the previous analysis[4–6] of their stability in the plane of the parameters Q_1 and Q_2.

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